DO NOW

Differentiate: $f(x) = \frac{1}{5}(x^2+1)^5 + 3$

5.5 Integration by Substitution

Recall the chain rule for differentiation:

$$\frac{d}{dx}\left[F(g(x))\right] = F'(g(x))\, g'(x)\, dx$$

$$\frac{d}{dx}\left[F(u)\right] = F'(u)\, u'\, du$$

For example: $f(x) = \frac{1}{5}(x^2+1)^5 + 3$

$$f'(x) = \frac{1}{5}(5)(x^2+1)^4(2x) + 0$$

$$f'(x) = (x^2+1)^4(2x)$$

$$\frac{dy}{dx} = (x^2+1)^4(2x)$$

Now find the following integral:

$$\int (x^2+1)^4 (2x)\, dx$$

$u = x^2 + 1$

$du = 2x\,dx$

$$\int u^4\, du$$

$\frac{1}{5}u^5 + C$ * sub u back in

$\frac{1}{5}(x^2+1)^5 + C$

Antidifferentiation of a Composite Function

Let g be a function whose range is an interval I, and let f be a function that is continuous on I. If g is differentiable on its domain and F is an antiderivative of f on I, then:

$$\int f(g(x))\, g'(x)\, dx = F(g(x)) + C$$

u-substitution

$$\int f(u)\, du = F(u) + C$$

Examples:

1. $\int (1-2x^2)^3 (-4x)\, dx$

$u = 1 - 2x^2$

$du = -4x\,dx$

$$\int u^3\, du$$

$\frac{1}{4}u^4 + C$

$\frac{1}{4}(1-2x^2)^4 + C$

2. $\int 2x\sqrt{1+x^2}\, dx$

$u = 1 + x^2$

$du = 2x\,dx$

$$\int u^{1/2}\, du$$

$\frac{2}{3}u^{3/2} + C$

$\frac{2}{3}(1+x^2)^{3/2} + C$

3. $\int 5x^4(1+x^5)^3\,dx$

$u = 1+x^5$
$du = 5x^4\,dx$

$\int u^3\,du$

$\frac{1}{4}u^4 + C$

$\frac{1}{4}(1+x^5)^4 + C$

4. $\int x^4(1+x^5)^3\,dx$

$u = 1+x^5$
$du = 5x^4\,dx$
$\frac{du}{5} = x^4\,dx$

$\int u^3 \frac{du}{5}$

$\frac{1}{5}\int u^3\,du$

$\frac{1}{5}\cdot\frac{1}{4}u^4 + C$

$\frac{1}{20}(1+x^5)^4 + C$

5. $\int x(4x^2+3)^3\,dx$

$u = 4x^2+3$
$du = 8x\,dx$
$\frac{du}{8} = x\,dx$

$\int u^3 \frac{du}{8}$

$\frac{1}{8}\int u^3\,du$

$\frac{1}{8}\cdot\frac{1}{4}u^4 + C$

$\frac{1}{32}(4x^2+3)^4 + C$

6. $\int \frac{x^3}{\sqrt{x^4+16}}\,dx$

$u = x^4+16$
$du = 4x^3\,dx$
$\frac{du}{4} = x^3\,dx$

$\int x^3(x^4+16)^{-1/2}\,dx$

$\int u^{-1/2}\frac{du}{4}$

$\frac{1}{4}\int u^{-1/2}\,du$

$\frac{1}{4}(2u^{1/2}) + C$

$\frac{1}{2}(x^4+16)^{1/2} + C$

$\frac{\sqrt{x^4+16}}{2} + C$

HOMEWORK

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